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Breathing Vacuum Bubbles in Five-Dimensional Gauss-Bonnet Gravity

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Abstract

We study the dynamics of bubble wall in five-dimensional Gauss-Bonnet gravity in the thin-wall approximation. The motion of the domain wall is determined by the generalized Israel junction condition. We find a solution where the wall of true vacuum bubble is oscillating, the breathing bubbles. We briefly comment on how this breathing vacuum bubble can affect the analysis of the string theory landscape.

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1 Introduction

In the theory with multiple meta-stable vacua, string theory for example, our universe is very likely to tunnel from other vacua with higher cosmological constants. The process is similar to the first order phase transition, where the bubble with different vacuum energy will nucleate from the original vacuum. When couple to the gravity, the vacuum with positive energy will expand exponentially. This leads to an interesting cosmology of eternal inflation [1].

In Einstein gravity, the geometry and dynamics of the bubble nucleation have been studied extensively, see [3] and references therein. The motion of the bubble wall, in the thin wall approximation is determined by the Israel junction condition [2]. There are various kinds of solutions, but they fall into two main categories. When the wall tension can't compensate the energy occupied by the bubble's volume, it will eventually shrink to zero size. Otherwise the bubble wall will eventually expand and quickly approach the speed of light.

However, these are not the most general solutions. The bubble walls can oscillate and neither shrink to zero size nor asymptote to the light cone. This kind of breathing bubble solution, in Einstein gravity has been found in the paper [4]. The solution in their case requires the bubble wall to have non-standard equation of state. Similar solution is also found in the study of brane-world cosmology, where our universe lives on the thin brane embedded in the higher dimension space-time. The Israel junction condition here determined the scale factor of the brane-world. In [5], the author found that, in order to get the breathing type solution the brane must be embedded in the Anti-de Sitter space.

The Gauss-Bonnet gravity is a particular higher derivative correction to Einstein gravity, which leaves the equation of motion remain second order. It is topological in four dimension and will not affect the dynamics. In this paper we instead study the bubble dynamics in the five-dimensional(5D) Gauss-Bonnet gravity. The energy momentum tensor of the bubble wall is assumed to be the form of ordinary scalar field. In many cases the solution is qualitatively the same as in Einstein gravity. However in a wide range of parameter space, we find the breathing bubble solution for the true vacuum bubble with positive vacuum energy. At first sight, it may seem strange to consider the bubble in five-dimensional Gauss-Bonnet gravity. However in the UV complete theory of gravity. It is generic to have higher derivative corrections to Einstein gravity. When we consider the evolution of the early universe, the effect of higher derivative terms can play an important role. Moreover, landscape of vacua in string theory [6] also contains those with effectively space-time dimension higher than four. The transition between bubbles with different dimension is also possible [7]. There is a possibility that our universe tunnelled from such higher dimensional bubbles. Thus it may be too hasty to restrict ourself on 4D Einstein gravity. For example, it needs to be more careful when discussing the measure problem, the problem to assign the probability to different vacuum bubbles.(See [8] for review). As it usually assumes that the bubble wall approaches the light cone.

In section two, we will first review the bubble wall dynamics in 5D Einstein gravity. Then we turn to the 5D Gauss-Bonnet gravity and find the breathing bubbles. We conclude in section three and point out the possible effect of breathing bubble solution on the measure problem. In Appendix

A, we discuss the difficulty of the thin wall approximation in general higher derivative gravity other than Gauss-Bonnet gravity. However for showing the effects of higher derivative corrections, we believe that the Gauss-Bonnet gravity is sufficient.

2 Bubble dynamics in 5D Gauss-Bonnet gravity

In this section, we first study the bubble dynamics in 5D Einstein gravity. The solutions will not be much different from 4D Einstein gravity if we assume the spherical symmetry. Nevertheless the results can then be used to compare with the ones from the 5D Gauss-Bonnet gravity. We then turn to bubble dynamics in 5D Gauss-Bonnet gravity. One complication is that the analogy of particle dynamics in 1D effective potential is not obvious in the Gauss-Bonnet counterpart. Here we use the phase space analysis to study the bubble wall trajectories.

2.1 Vacuum bubble dynamics in 5D Einstein gravity

We start with the 5D Einstein-Hilbert action

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g}\, [R - 2\Lambda] \tag{2.1}$$

where κ^2 is the 5D Newton constant, and Λ is the cosmological constant. Assuming spherical symmetry for both the bulk space-time and the bubble wall, we can solve the Einstein equation and get the bulk metric solution in the following form

$$ds^2 = -f_{in,out}(r)dt^2 + f_{in,out}(r)^{-1}dr^2 + r^2 d\Omega_3^2 \tag{2.2}$$

where $f_{in,out}(r)$'s are the harmonic function for the inside and outside of the bubble.

Given the inside and outside forms of the metric, the bubble wall dynamics is dictated by the Israel junction condition [2], which can be derived from Einstein equation and takes the form as

$$[K_{ab} - K h_{ab}]^+_- = -\kappa^2 S_{ab} \tag{2.3}$$

where K_{ab} is the extrinsic curvature of the bubble wall, and S_{ab} is the stress tensor of matters on the bubble wall. According to spherical symmetry, we can choose the induced metric on the bubble wall in the following form

$$ds^2 = -d\tau^2 + r^2(\tau) d\Omega_3^2. \tag{2.4}$$

Then, the stress tensor of the wall takes the form

$$S_{ab} = -\sigma h_{ab} \tag{2.5}$$

where σ can be understood as the tension of the wall.

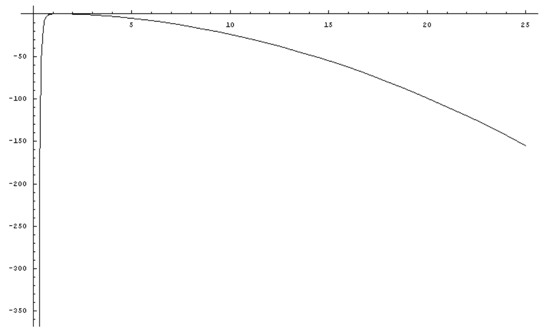

Figure 1: Effective potential for true vacuum bubble in Einstein gravity.

Choosing the gauge $t = \tau$, and plugging the wall metric into the Israel junction condition to get the dynamical equation for the bubble radius $r(t)$. Due to the spherical symmetry, the only independent equation (the tt component only) takes the form

$$-\frac{\kappa^2\sigma}{3}r = \sqrt{\dot{r}^2 + f_{out}} - \sqrt{\dot{r}^2 + f_{in}}\ , \tag{2.6}$$

where $\dot{r} := \frac{dr}{dt}$. This takes the same form as for the 4D Einstein gravity.

To be more specific, we now consider the dynamics of the true vacuum bubble, that is

$$f_{in} = 1\ , \qquad f_{out} = 1 - \frac{\Lambda}{6}r^2 - \frac{\mu}{r^2}\ , \tag{2.7}$$

which describe an Minkowski bubble surrounded by the de Sitter space, and μ is the mass of the bubble measured by an asymptotic outside observer. After some rearrangement, (2.6) can be put into form of particle dynamics

$$\frac{1}{2}\dot{r}^2 + V_{eff} = 0 \tag{2.8}$$

with an effective potential as

$$V_{eff} = \frac{1}{2} - \frac{9}{8}\frac{\left((\frac{\kappa^4\sigma^2}{9} - \frac{\Lambda}{6})r^2 + \mu/r^2\right)^2}{\kappa^4\sigma^2 r^2} \tag{2.9}$$

The form of the effective potential is shown in Figure 1. This is quite similar to case for 4D Einstein gravity.

By inspecting the form of the effective potential, it is easy to see there are three kinds of solutions of (2.8) and (2.9) characterized by the behaviors of $r(t)$. They are bounded, bounce and monotonic ones, which correspond to the shrinking, bouncing and ever-expanding bubbles, respectively. Again, these solutions are similar to the ones in 4D Einstein gravity.

One can also change the particle dynamics to the Hamilton formalism by regarding (2.8) as a Hamiltonian constraint, i.e., $H = 0$. By tuning the initial conditions, one can then numerically characterize the above 3 different solutions in the phase space ($q = r, p = \dot{r}$) as shown in Figure 2.It seems redundant to numerically solve the bubble wall dynamics in the Hamilton's formalism. However, it turns out to be quite essential for the case of Gauss-Bonnet gravity, in which particle dynamics is no longer quadratic and is hard to visualize the solution by just inspecting the effective potential.

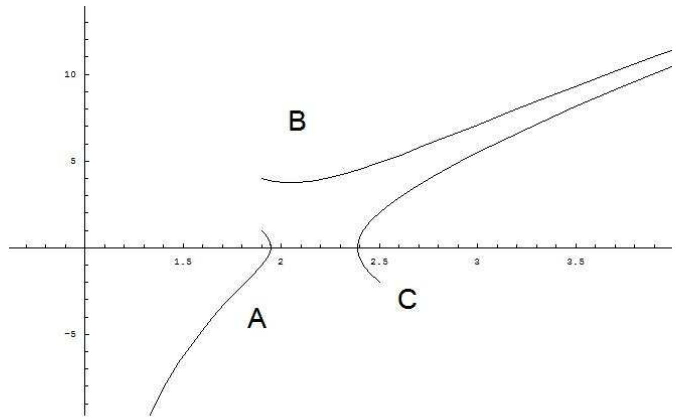

Figure 2: True vacuum bubble solutions in phase space for 5D Einstein gravity. A: bounded solution. B: monotonic solution. C: bounce solution.

2.2 Junction condition of 5D Gauss-Bonnet gravity

We now move to the bubble dynamics in the 5D Gauss-Bonnet gravity by starting with the action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}[R - 2\Lambda + \alpha\mathcal{L}_{GB}] \tag{2.10}$$

where α is the Gauss-Bonnet coupling and the form of Gauss-Bonnet term is

$$\mathcal{L}_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \ . \tag{2.11}$$

Due to the Gauss-Bonnet term, the junction condition for the bubble wall becomes more involved as following [9], and it takes the form as

$$[K_{ab} - Kh_{ab}]^+_- + 2\alpha[3J_{ab} - Jh_{ab} + 2P_{acdb}K^{cd}]^+_- = -\kappa^2 S_{ab} \tag{2.12}$$

where

$$J_{ab} = \frac{1}{3}(2KK_{ac}K^c_b + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{db} - K^2K_{ab}) \tag{2.13}$$

and

$$P_{abcd} = R_{abcd} + 2R_{b[c}g_{d]a} - 2R_{a[c}g_{b]b} + Rg_{a[c}g_{d]b} \ . \tag{2.14}$$

Again assuming spherical symmetry for the bubble wall with the bulk and wall metrics (2.2) and (2.4), respectively; then tt component of the junction condition in the $t = \tau$ gauge can be reduced to

$$-\kappa^2\sigma = \Big[3\frac{\sqrt{\dot r^2 + f(r)}}{r} - 2\alpha\Big(2\frac{\left(\sqrt{\dot r^2 + f(r)}\right)^3}{r^3} - 6(1+\dot r^2)\frac{\sqrt{\dot r^2 + f(r)}}{r^3}\Big)\Big]^+_-$$

where we have used (2.5) for the stress tensor of the matter on the wall. It is clear that the ”particle dynamics” for $r(t)$ based on (2.15) is no longer quadratic, and in general it can be transformed into the following form

$$X(r)\dot{r}^2 + Y(r)\dot{r}^4 + Z(r)\dot{r}^6 + V(r) = 0$$

where the exact forms of the functions $X(r)$, $Y(r)$, $Z(r)$ and $V(r)$ depend on $f_{in}(r)$ and $f_{out}(r)$.

Therefore, we can no longer use quadratic particle dynamics analogue to characterize the solutions of $r(t)$, instead it is far easier to use phase space analysis. Let us parameterize the phase space as $(q = r, p = \dot{r})$, and the Hamiltonian be

$$H = X(q)p^2 + Y(q)p^4 + Z(q)p^6 + V(q) \ , \tag{2.15}$$

then the Hamilton's equations are

$$\begin{aligned} \dot{q} &= 2X(q)p + 4Y(q)p^3 + 6Z(q)p^5 \ , && (2.16) \\ -\dot{p} &= \frac{\partial X}{\partial q}p^2 + \frac{\partial Y}{\partial q}p^4 + \frac{\partial Z}{\partial q}p^6 + \frac{\partial}{\partial q}V \ . && (2.17) \end{aligned}$$

Given $f_{in,out}(r)$, we will then numerically solve these equations to characterize the solutions.

To find the explicit forms of X, Y, Z and V, we assume the spherical symmetry so that bulk and wall metrics again take the forms of (2.2) and (2.4), respectively. However, the inside and outside harmonic functions solved the field equations of Gauss-Bonnet gravity are the Boulware-Deser-Schwarzschild-de Sitter ones [10], that is,

$$f_{in}(r) = 1 + \frac{r^2}{4\alpha}(1 - \sqrt{1 + \frac{4\alpha\lambda}{3}}) \tag{2.18}$$

where λ is the cosmological constant inside the bubble; and

$$f_{out}(r) = 1 + \frac{r^2}{4\alpha}(1 - \sqrt{1 + \frac{4\alpha\Lambda}{3} + \frac{8\alpha\mu}{r^4}}) \tag{2.19}$$

where Λ is the cosmological constant outside the bubble, and μ is the mass of the bubble. If λ is zero, this is the true vacuum bubble. On the other hand, the false vacuum bubble has $\Lambda = 0$.

Given (2.18) and (2.19), the functions X, Y, Z and V appearing in (2.15) for Hamilton formalism are

$$\begin{aligned} X(r) &= -32\alpha^2\Lambda^2 + 384\alpha k^2 + 64\alpha^2\Lambda\lambda - 32\alpha^2\lambda^2 - \frac{1152\alpha^{2\mu^2}}{r^8} - \frac{288\alpha\mu^2}{r^6} - \frac{384\alpha^2\Lambda\mu}{r^4} + \frac{384\alpha^2\lambda\mu}{r^4} + \frac{768\alpha^2k^2}{r^2} \\ &\quad - \frac{96\alpha\Lambda\mu}{r^2} - \frac{48\alpha k^2\mu}{r^2} + \frac{96\alpha\lambda\mu}{r^2} - 8\alpha\Lambda^2r^2 + 36k^2r^2 - 8\alpha\Lambda k^2r^2 + 16\alpha\Lambda\lambda r^2 - 8\alpha\lambda k^2r^2 - 8\alpha\lambda^2r^2 \\ &\quad + \Big(-2r^2\Lambda - \frac{8}{3}r^2\alpha\Lambda^2 + 2r^2\lambda + \frac{8}{3}r^2\alpha\lambda\Lambda - \frac{12\mu}{r^2} - \frac{32\alpha\Lambda\mu}{r^2} - \frac{96\alpha\mu^2}{r^6} + \frac{16\alpha\lambda\mu}{r^2}\Big)\sqrt{1 + \frac{4\alpha\Lambda}{3} + \frac{8\alpha\mu}{r^4}} \\ &\quad + \Big(\frac{12\mu}{r^2} + \frac{16\alpha\lambda\mu}{r^2} + 2\Lambda r^2 - 2\lambda r^2 + \frac{8}{3}\alpha\Lambda\lambda r^2 - \frac{8}{3}\alpha\lambda^2r^2\Big)\sqrt{1 + \frac{4\alpha\lambda}{3}} \ , && (2.20) \end{aligned}$$

$$Y(r) = -16\alpha^2\Lambda^2 + 192\alpha k^2 + 32\alpha^2\Lambda\lambda - 16\alpha^2\lambda^2 + \frac{768\alpha^2 k^2}{r^2} - \frac{192\alpha^2\Lambda\mu}{r^4} + \frac{192\alpha^2\lambda\mu}{r^4} - \frac{576\alpha^2\mu^2}{r^8}\ , \tag{2.21}$$

$$Z(r) = \frac{256\alpha^2 k^2}{r^2}\ , \tag{2.22}$$

$$\begin{aligned} V(r) \quad = \quad & -16\alpha^2\Lambda^2 + 192\alpha k^2 - \frac{3\mu}{2\alpha} + 32\alpha^2\Lambda\lambda - 16\alpha^2\lambda^2 - 16\Lambda\mu - \frac{8\alpha\Lambda^2\mu}{3} - 12k^2\mu + 12\lambda\mu - \frac{576\alpha^2\mu^2}{r^8} \\ & - \frac{32\alpha\mu^3}{r^8} - \frac{288\alpha\mu^2}{r^6} - \frac{192\alpha^2\Lambda\mu}{r^4} + \frac{192\alpha^2\lambda\mu}{r^4} - \frac{48\mu^2}{r^4} - \frac{16\alpha\Lambda\mu^2}{r^4} + \frac{256\alpha^2k^2}{r^2} - \frac{96\alpha\Lambda\mu}{r^2} - \frac{48\alpha k^2\mu}{r^2} \\ & + \frac{96\alpha\lambda\mu}{r^2} - 8\alpha\Lambda^2r^2 + 36k^2r^2 - 8\alpha\Lambda k^2r^2 + 16\alpha\Lambda\lambda r^2 - 8\alpha k^2\lambda r^2 - 8\alpha\lambda^2r^2 - \frac{r^4}{8\alpha^2} \\ & - \frac{\Lambda r^4}{\alpha} - \frac{4}{3}\Lambda^2r^4 - \frac{4}{27}\alpha\Lambda^3r^4 + \frac{k^2r^4}{\alpha} - 2\Lambda k^2r^4 - k^4r^4 - \frac{\lambda r^4}{4\alpha} + 2\Lambda\lambda r^4 - 2k^2\lambda r^4 - \frac{4}{3}\lambda^2r^4 - \frac{4}{27}\alpha\lambda^3r^4 \\ & + \Big(-\frac{3\mu}{\alpha} - 8\alpha\Lambda - 4k^2\mu + 4\lambda\mu - \frac{96\alpha\mu^2}{r^6} - \frac{24\mu^2}{r^4} - \frac{12\mu}{r^2} - \frac{32\alpha\Lambda\mu}{r^2} + \frac{16\alpha\lambda\mu}{r^2} - 2\Lambda r^2 - \frac{8}{3}\alpha\Lambda^2r^2 \\ & + 2\lambda r^2 + \frac{8}{3}\alpha\Lambda\lambda r^2 - \frac{\Lambda r^4}{2\alpha} - \frac{2}{3}\Lambda^2r^4 - \frac{k^2r^4}{2\alpha} - \frac{2}{3}\Lambda k^2r^4 + \frac{\lambda r^4}{2\alpha} + \frac{2}{3}\Lambda\lambda r^4\Big)\sqrt{1 + \frac{4\alpha\Lambda}{3} + \frac{8\alpha\mu}{r^4}} \\ & + \Big(\frac{3\mu}{\alpha} + 4\lambda\mu + \frac{12\mu}{r^2} + \frac{16\alpha\lambda\mu}{r^2} + 2\Lambda r^2 - 2\lambda r^2 + \frac{8}{3}\alpha\Lambda\lambda r^2 - \frac{8}{3}\alpha\lambda^2r^2 \\ & + \frac{\Lambda r^4}{2\alpha} - \frac{k^2r^4}{2\alpha} - \frac{\lambda r^4}{2\alpha} + \frac{2}{3}\Lambda\lambda r^4 - \frac{2}{3}k^2\lambda r^4 - \frac{2}{3}\lambda^2r^4\Big)\sqrt{1 + \frac{4\alpha\lambda}{3}} \\ & + \Big(\frac{\mu}{\alpha} + \frac{4}{3}\lambda\mu + \frac{r^4}{8\alpha^2} + \frac{\Lambda r^4}{6\alpha} + \frac{\lambda r^4}{6\alpha} + \frac{2}{9}\Lambda\lambda r^4\Big)\sqrt{1 + \frac{4\alpha\lambda}{3}}\sqrt{1 + \frac{4\alpha\Lambda}{3} + \frac{8\alpha\mu}{r^4}}\ , \end{aligned} \tag{2.23}$$

where the re-scaled tension parameter

$$k := \kappa^2\sigma. \tag{2.24}$$

These forms are so complicated so that we can only rely on the numerical analysis on the phase space to characterize various kind of solutions by tuning the parameters and initial conditions.

2.2.1 Breathing true vacuum bubble

We first consider the dynamics of true vacuum bubble, which could be relevant to the global picture of string landscape. To implement numerical analysis of the Hamilton equations (2.16) and (2.17), we set $\Lambda = 1$, $\lambda = 0$, $\mu = 1$ and $\alpha = 0.01$ in (2.20)-(2.23). For generic values of k, we will as usual have bounded, bounce and monotonic solutions, but for some regime of k we have the breathing bubble solution, i.e., the size of the bubble is pulsating/breathing so that the phase space trajectory is periodic. The general numerical solutions on phase space are illustrated in Figure 3.

To examine more closely on the breathing solution, we tune the re-scaled tension parameter k for a given some initial condition, and find that there is a critical value of k, lower than that there will exist the breathing solution. The dependence on the initial condition of bubble's phase space trajectory is shown in Figure 4.

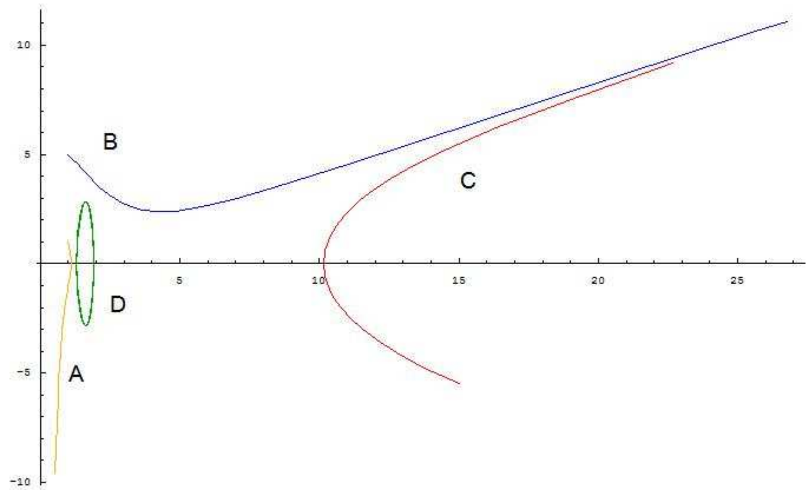

Figure 3: True vacuum bubble solutions on phase space for 5D Gauss-Bonnet gravity. A: bounded solution. B: monotonic solution. C: bounce solution. D: breathing solution.

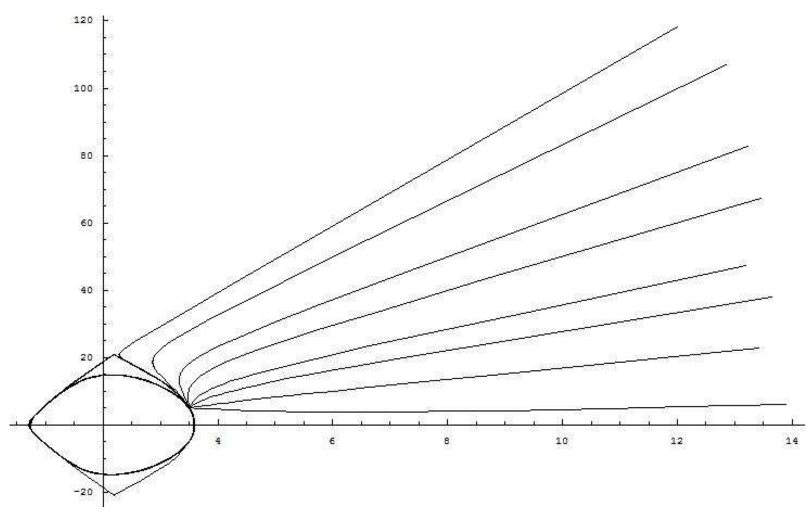

Figure 4: Tuning the re-scaled tension parameter k for breathing bubble. We fix $\Lambda = 1$, $\mu = 1$, $\alpha = 0.01$, and the initial condition $p = 5$ and $q = 3.5$ at which all the lines intersect in the phase space diagram. The solution corresponding to the almost horizontal line has $k = 1$, and then the value of k decreases for the lines arranged counter-clockwise until $k = 0.0253$ below that there exists the breathing solution, i.e., the elliptical shape ones.

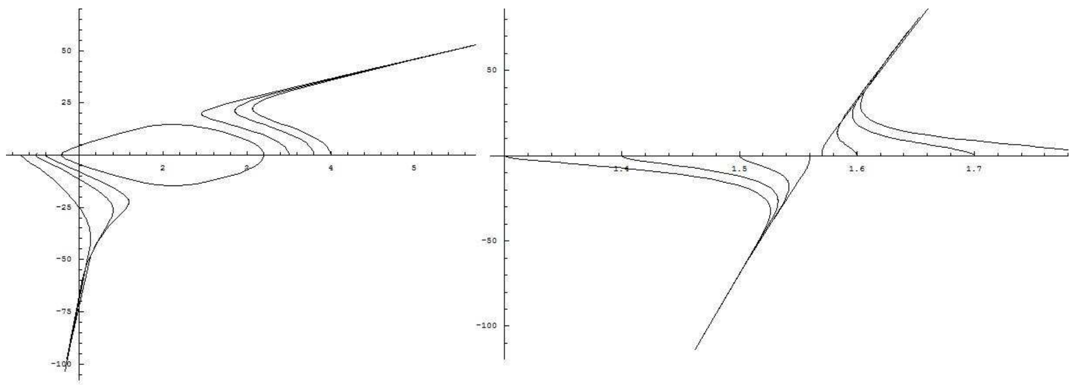

Figure 5: Phase space diagram for tuning the initial conditions for the true (Left) and the false (Right) vacuum bubbles. On the left, we set $\alpha = 0.01$, $k = 0.027$, $\mu = 1$, $\Lambda = 1$ and $\lambda = 0$. On the right, we set $\alpha = 0.01$, $k = 0.001$, $\mu = 1$, $\Lambda = 0$ and $\lambda = 1$. We see there is a breathing solution for the true vacuum bubble, but not for the false one.

2.2.2 No breathing false vacuum bubble

We can also consider the dynamics of the false vacuum by choosing $\Lambda = 0$ and $\lambda = 1$ for the harmonic functions (2.18) and (2.19). Again, solving the Hamilton equations (2.16)-(2.17) with the set of functions in (2.20)-(2.23) in terms of the new harmonic functions, we find that there is no breathing solution. For illustration, in Figure 5 we juxtapose the phase space diagrams by tuning the initial conditions for the true and false vacuum bubbles.

We can also consider the cases with both Λ and λ non-zero, then it is more close to true vacuum bubble if $\Lambda > \lambda$, and more close to false vacuum bubble if $\Lambda < \lambda$. From our numerical studies with quite extensive ranges for both parameters and initial conditions, we find that there exist breathing bubble solutions for the former case, but not for the latter. We may then tend to state that there are breathing vacuum bubble solutions in 5D Gauss-Bonnet gravity if the cosmological constant inside the bubble is smaller than the one outside, but not vice versa.

3 Conclusion and the implications

The higher derivative terms in Einstein-Hilbert action can be ignored when the length scale considered is much smaller than the curvature radius. Thus there is no reason to dwell on the Einstein gravity when we consider the global picture of the universe. In this case it will be important to exam the effect of the higher derivative corrections. One new feature we found in this paper is the breathing bubble solution in 5D Gauss-Bonnet gravity. In particular, we don't need the exotic matter for the bubble wall and this kind of solutions exist for true vacuum bubble with zero or positive vacuum energy. One possible implication is on the measure problem of eternal inflation. In order to do the sensible statistics on bubble distribution, we need to cut-off the infinite space-time and infinite number of bubbles properly. One way of doing this is to count the bubbles smaller than a fixed scale factor [11]. However usually people assume the bubble nucleating at vacuum

with Hubble constant H will have co-moving radius H^{-1} asymptotically. This is not the case if there are breathing bubbles whose co-moving sizes are vary with time and depend on critical bubble size. Including these bubbles will change the evolution equation for the fraction of volume occupied by a particular bubble. One may also ask what happens if a growing bubble nucleates inside a breathing bubble. The bubble walls can collide and generate the gravitational wave. This can have the observational consequences if our universe goes through such kind of bubbles in the past evolution. We leave these problems for the future studies.

Acknowledgements

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A Difficulty for thin-wall vacuum bubbles in higher derivative gravity

One difficulty of studying bubble dynamics in arbitrary higher derivative gravity is due to the appearance of higher order singularity when assuming the thin wall approximation. In [12], they try to avoid this difficulty by assuming the discontinuity of extrinsic curvature only appears in higher order derivative. However we don't have solutions satisfing this criterion yet. In this appendix, we will derive the junction conditions modified by higher order corrections and argue that in general they are not the consistent sets of equations.

A Lagrangian including higher order corrections derived from high energy theory like string theory may have the form

$$L = R - 2\Lambda + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$$

A special case for second order gravity is $\alpha = \gamma = 1, \beta = -4$ which is the Einstein-Gauss-Bonnet gravity. In four dimensions these terms are topological invariant which does not contribute to the equation of motion. For simplicity, we consider $L^{(2)} = R - 2\Lambda + \alpha R^2$ first and show that thin wall approximation is not reliable. The field equation from variation of $\sqrt{g}L$ is [12]

$$\sigma_{\mu\nu} = (1 + 2\alpha R) G_{\mu\nu} + \frac{1}{2}\alpha R^2 g_{\mu\nu} - 2\alpha D_\mu D_\nu R + 2\alpha g_{\mu\nu} \Box R^2$$

$$\sigma_{ij} = \Lambda h_{ij} + G_{ij} + 2\alpha R G_{ij} + \alpha \frac{1}{2} R^2 h_{ij} + 2\alpha[-\overline{D}_i \overline{D}_j R + K_{ij}\partial_y R + h_{ij}(\partial_{yy} R + \overline{\Box} R - K\partial_y R)]$$

In order to extract the discontinuity from equation of motion, we assume the metric is continuous at the wall, but it has a kink. Its first derivative has a step function discontinuity and its second derivative has a delta function term [13]

$$h_{ij}(y) = h^-_{ij}\theta(-y) + h^+_{ij}\theta(y)$$

$$\frac{\partial h_{ij}(y)}{\partial y} = \frac{\partial h^-_{ij}(y)}{\partial y}\theta(-y) + \frac{\partial h^+_{ij}(y)}{\partial y}\theta(y)$$

$$\frac{\partial^2 h_{ij}(y)}{\partial^2 y} = \frac{\partial^2 h^-_{ij}(y)}{\partial^2 y}\theta(-y) + \frac{\partial h^+_{ij}(y)}{\partial y}\theta(y) + (\frac{\partial h^-_{ij}(y)}{\partial y} + \frac{\partial h^+_{ij}(y)}{\partial y})\delta(y)$$

In Einstein's gravity, substitute (24) into $G_{ij} = \kappa^2 T_{ij}$ and match the delta function term, so that we can get the Israel junction condition. In order to get the junction condition for the R^2 correction gravity, we need to work out following quantity

$$\begin{aligned} RG_{ij} &= 2\partial_y K \partial_y (K_{ij} - K h_{ij}) + 2\partial_y K \Big(2K^l_i K_{lj} + \partial_y K_{lj} - K K_{ij} + \frac{1}{2} h_{ij}(TrK^2 + K^2) + \bar{G}_{ij}\Big) \\ &+ \partial_y (K_{ij} - K h_{ij})(-TrK^2 - K^2 + \bar{R}) \end{aligned} \tag{A.1}$$

$$R^2 = [2\partial_y K - TrK^2 - K^2 + \bar{R}]^2 \tag{A.2}$$

$$\partial_y R K_{ij} = [2\partial_{yy} K - \partial_y TrK^2 - \partial_y K^2 + \partial_y(\bar{R})]K_{ij} \tag{A.3}$$

$$\partial_{yy} R = 2\partial_{yyy} K - \partial_{yy} TrK^2 - \partial_{yy} K^2 + \partial_{yy}(\bar{R}) \tag{A.4}$$

$$K\partial_y R = K(2\partial_{yy} K - \partial_y TrK^2 - \partial_y K^2 + \partial_y(\bar{R}) \tag{A.5}$$

We consider that induced metric h_{ij} is continuous across the domain wall, but discontinuous across the wall. However terms like R^2 would give $\delta_(y)^2$ and $\partial_y R$ would give $\partial_y \delta(y)$ and $\partial_{yy} R$ would give $\partial_{yy}\delta(y)$. Those more singular terms are not well defined. One may require higher continuity of the induced metric h_{ij}, but this would make the equation of the domain wall trivial. We cannot assume this condition in our case. We propose to include more singular source terms on the wall to see if the equation of motion can be solved consistently. Introducing source terms

$$T^{\mu\nu} = S^{\mu\nu}(x^i)\delta(y) + U^{\mu\nu}(x^i)\delta(y)^2 + V^{\mu\nu}(x^i)\partial_y\delta(y) + W^{\mu\nu}(x^i)\partial_{yy}\delta(y)$$

Due to the conservation law$\nabla_\nu T^{\mu\nu} = 0$

$$\begin{aligned} \nabla_\nu T^{i\nu} &= (\nabla_j S^{ij} + 2K^i_j S^{jy} + TrKS^{iy})\delta(y) + S^{iy}\partial_y\delta(y) \\ &+(\nabla_j U^{ij} + 2K^i_j U^{jy} + TrKU^{iy})\delta(y)^2 + U^{iy} 2\delta(y)\partial_y\delta(y) \\ &+(\nabla_j V^{ij} + 2K^i_j V^{jy} + TrKV^{iy})\partial_y\delta(y) + V^{iy}\partial_{yy}\delta(y) \\ &+(\nabla_j W^{ij} + 2K^i_j W^{jy} + TrKW^{iy})\partial_{yy}\delta(y) + W^{iy}\partial_{yyy}\delta(y) \\ &= 0 \end{aligned}$$

We should have the following constraints.

$$\begin{aligned} \nabla_j S^{ij} + 2K^i_j S^{jy} + TrKS^{iy} &= 0 \\ \nabla_j U^{ij} + 2K^i_j U^{jy} + TrKU^{iy} &= 0 \\ S^{iy} + \nabla_j V^{ij} + 2K^i_j V^{jy} + TrKV^{iy} &= 0 \\ U^{iy} &= 0 \\ \nabla_j W^{ij} + 2K^i_j W^{jy} + TrKW^{iy} + V^{iy} &= 0 \\ W^{iy} &= 0 \end{aligned}$$

Matching the left hand side and right hand side of equation for the first and second derivatives of the delta function and the square of the delta function respectively. It gives four equations, where one being proportional to $\delta(y)$ is rather cumbersome, so we neglect it, but it will not affect our result

$$\partial_{yy}\delta(y) \longrightarrow -2\alpha h^{ij+}(\partial h^{+}_{ij} - \partial h^{-}_{ij})h_{ij} = W_{ij}(x^i)$$

$$\begin{aligned}\delta(y)^2 \longrightarrow &\frac{\alpha}{2}h^{+kl}(\partial_y h^{+}_{kl} - \partial_y h^{-}_{kl})(\partial_y h^{+}_{ij} - \partial_y h^{-}_{ij}) + \frac{\alpha}{2}[h^{+kl}(\partial_y h^{+}_{kl} - \partial_y h^{-}_{kl})h^{+mn}(\partial_y h^{+}_{mn} - \partial_y h^{-}_{mn})]h_{ij} \\ &+ \frac{\alpha}{4}[h^{kl+}(\partial_y h^{+}_{kl} - \partial_y h^{-}_{kl})h^{+mn}(\partial_y h^{+}_{mn} - \partial_y h^{-}_{mn})(\partial_y h^{+}_{ij} + \partial_y h^{-}_{ij})] + \frac{\alpha}{2}[h^{+ij}(\partial_y h^{+}_{ij} - \partial_y h^{-}_{ij})]^2 h_{ij} = U_{ij}\end{aligned}$$

$$\begin{aligned}\partial_y\delta(y) \longrightarrow &\frac{\alpha}{2}[h^{+kl}\partial_y h^{+}_{ij}\partial_y h^{+}_{kl} + h^{+kl}\partial_y h^{-}_{ij}\partial_y + h^{+ij}(h^{+kl}\partial_y h^{+}_{ij}\partial_y h^{+}_{kl} + h^{+kl}\partial_y h^{-}_{ij}\partial_y + h^{-}_{kl})h_{ij} \\ &- 2(\partial_y h^{+ij}\partial_y h^{+}_{ij} + \partial_y h^{+ij}\partial_y h^{-}_{ij} + h^{+ij}\partial_y^2 h^{+}_{ij} + h^{+ij}\partial_y h^{-}_{ij} + \partial_y h^{+ij}(\partial_y h^{+}_{ij} - \partial_y h^{-}_{ij}) \\ &+ h^{+ij}(\partial_y h^{+}_{ij} - \partial_y h^{-}_{ij}) + \partial_y h^{+ij}\partial_y h^{+}_{ij} + h^{+ij}\partial_y^2 h^{+}_{ij} - \partial_y h^{-ij}\partial_y h^{-}_{ij} - h^{-ij}\partial_y h^{-}_{ij})h_{ij} \\ &+ (h^{+ik}h^{+jl}(\partial_y h^{+}_{ij}\partial_y h^{+}_{kl} - \partial_y h^{-}_{ij}\partial_y h^{-}_{kl}) + h^{+ij}h^{+kl}(\partial_y h^{+}_{ij}\partial_y h^{+}_{kl} - \partial_y h^{-}_{ij}\partial_y h^{-}_{kl}))h_{ij}] = V_{ij}\end{aligned}$$

We can see that these three equations are very different and cannot be solved simultaneously. It shows the breakdown of the thin wall approximation. Instead, we should consider finite thickness of the domain wall to avoid severe singular behavior.

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